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The Evolution of Coalition Game Strategies

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Abstract

This research explores two and three player coalition game strategies. Specifically, the paper explores a method for automating play among different players via computer, each with predefined strategies. Furthermore, the research investigates how different strategies perform by placing the players into virtual evolutionary environments whereby the players receiving the most payoffs survive and increase in number. The paper describes several different structures of game negotiation that were implemented for the virtual game simulations and the advantages and disadvantages that they provide. Finally, some results and ideas for future work are discussed.

1. Introduction

If people are presented with the option to collaborate in order to save money or resources, will they choose to do so, and if so how will they choose to allocate the savings? This project sought to find answers to this type of question by simulating interactions between players with strategies that human's could choose to employ.

In game theory, a game consists of two or more players who interact in clearly defined ways resulting in clearly defined payoffs. The goal for this project was to explore these types of games and create automated computer playing strategies which could be played against one another in hopes of learning something about various strategies and how they might perform against one another. This paper will summarize the development of the game protocol, strategies, population dynamics, and attempts at using an evolutionary algorithm.

Early attempts were aimed at three player coalition games. A three player coalition game has a payoff for each coalition, which, if agreed upon, can be divided among the players in the coalition. If the players are named A, B, and C, then there would be a payoff for ABC, AB, AC, and BC. For all the games considered during this project, the individual player coalitions of simply A, B and C were considered to have a payoff of zero. In other words, the players only have something to gain if they work together.

The notation w(ABC) = 30 can be used to indicate that the coalition ABC has a payoff of 30. Similarly w(AB) = 24 indicates that the coalition AB has a payoff of 24. Likewise w(A)=0, w(B)=0, and w(C)=0 indicates that the payoff for a coalition of a single player would receive a payoff of zero.

To put this into context of concrete reality, consider the three players A, B, and C to be three cities that are working on building a highway to connect to a new city. If the three build the highway together they can save money, as opposed to each building their own highway and incurring the full cost of building. The full definition of such a game might be:

w(ABC) = 30	w(A) = 0
w(AB) = 24	w(B) = 0
w(AC) = 24	w(C) = 0
w(BC) = 18	

If all three work together the total savings is highest, but if A and B work together, the savings, as you can see above, might be different than the savings if A and C were to work together. In order to play the game, players must somehow reach an agreement about which players will work together and how they will divide the payoff that they receive by cooperating. If there are multiple players unwilling to reach an agreement, there might never be an agreement reached and so then all the players would receive a payoff of zero.

2. Coalition Game Simulation

Humans, when negotiating, are capable of negotiating between each other through very informal and arbitrary communication. In order to simulate negotiations between computer players, however, a rigid negotiation protocol was necessary. The first protocol actually used in game simulations functions in the following manner: players take turns, one at a time, in which they can make exactly one proposal to exactly one coalition (of two or three players). In this system, players can agree to another player's proposal by making the same exact proposal to the same coalition. Players are allowed to have at most one proposal offered for each coalition. This means that at any point player A can potentially have one proposal to AB, one to AC, and one to ABC. When a new proposal is made to a coalition where there currently is an offer, the new proposal replaces the old one. This can be thought of as one player trying to work a deal with just one other player if the coalition of three players doesn’t work, or vice versa. By having this protocol in place, problematic scenarios involving end game decisions were avoided.

In our negotiating procedure the game ends if all the players in a particular coalition have proposed identical proposals to this same coalition. It should be noted here, that the game only ends at this point in the case of two and three player games, because the smallest possible coalition size is two and in the two or three player games, subtracting two players (in a binding agreement) results in zero and one player which is insufficient to form a second coalition of cooperative players. In four or more player games, there would be the option of allowing the game to continue on, allowing other coalitions to form after the first.

Let the following notation indicate player A making a proposal to the coalition ABC with payoffs of 14 to A, 8 to B, and 8 to C.

A -> (14, 8, 8)

Similarly, this same notation can be used to define B making a proposal to the coalition BC with payoffs of 12 to B and 6 to C

B-> (null, 12, 6)

In general the notation

p->(x, y, z)

Means that player p proposes the value of x for player A, y for player B, and z for player C. If the value for a player is null, then they are not included in the coalition.

Thus, a sample negotiation may be something very simple like:

A -> (14, 8, 8)
B -> (14, 8, 8)
C -> (14, 8, 8)

Where A makes a proposal and then the other players immediately agree. Another negotiation may go something like this:

A -> (14, 8, 8)
B -> (null, 12, 6)
C -> (null, 11, 7)
A -> (13, 9, 7)
B -> (null, 11, 7)

In this case B and C decide to work together rather than include A in their negotiations. B suggests a 12 to 6 cut with C, but C makes a counter offer which B then accepts.

The other way that the game ends (in order to prevent infinite cycles of player turns) is that after a set number of safe rounds, danger rounds occur where after player takes a turn, if the game is not over, then a random chance determines if the game will continue or not. For instance, after three players each take three turns (nine total), there might be a 25% chance that the game will end after each successive turn. This prevents the game from going on forever and it forces players to try to cooperate or else they will gain nothing when the game is over.

Using this particular negotiating protocol, a player program was designed in order to implement different playing strategies which could be controlled through the use of setting different variables which dictated decision making procedures of the players. Early testing of these strategies consisted of only a couple basic variables. The idea behind this model of the playing strategy was that a player will have a baseline offer which they will absolutely accept. This may change from turn to turn, but on any given turn, if given a good enough offer, a player will definitely accept the offer if it ends the game and gives them at least their "minimum goal" payoff. This was one variable which could be assigned a decay rate, which meant that players would lower their standards as the game went on (or not if the minimum goal decay rate was zero). This would make sense because the longer the game goes on, the more likely the game is to end and the more desperate the players ought to be. Also, it is reasonable to assume that any given player would make proposals where they are getting more than their minimum goal payoff, because they would not want to have an offer accepted if they are not getting at least their minimum acceptable payoff. For simplicity, the proposal for the self was based as a fixed, higher percentage of the minimum acceptable payoff. For sake of simplicity the decay rate for the minimum goal was linear, and the proposal payoff percentage was static, but in principle there could be functions that would compute the appropriate values of the minimum acceptable level and proposal level. This ability to have a function determine the acceptance and proposal through functions allows this model to be perfectly generic in that any possible player strategy could be implemented in this framework. The one thing lacking perfect generality in this model of a player is in how the player will decide to divide the payoff between two players for three player coalition proposals and how to decide which coalition to propose to.

In order to handle any possible strategy that a player might implement, two more functions were implemented. One function is to generate a proposal given a coalition (and potentially a history or at least present state of the negotiation), and the other function was to evaluate the attractiveness of any potential proposal given the proposal and history or present state of the negotiation.

With a combination of these functions the playing strategies that could be implemented seem to include all reasonable choices. Using the attractiveness function, players can be biased towards negotiating with a particular player, towards a certain sense of fairness, jealousy or any other attribute of players or conditions within the game.

In the three player game, there is significant complexity in trying to determine how to generate proposals for the three player coalition. The increase in complexity over two player coalition proposals comes from the fact that the proposal must divide the payoff remaining after the proposer's share is removed between two players. A simple equal split might seem like an obvious answer, but there are many factors that could go into calculating how it should be split, including mathematical fairness properties and values as well as the previous proposals by the other players themselves.

For the evaluation of attraction, certain variables were assigned to reflect a bias towards certain criteria. Some of these factors included desiring: two player coalitions, three player coalitions, creating new proposals and copying (or agreeing with) other proposals. As will be shown later, assigning variables to these characteristics was done in hopes of evolving effective player strategies. These variables could then be mutated in order to generate new player strategies.

In order to compare or rank player strategies, games were simulated between players. Generally, there would be a set number of different player strategies and so a round robin tournament would be performed where each player would play every other player in every turn order. The turn rotation was a simple cycle of 1, 2, 3, 1, 2, 3 … so each player had to play in each position (1, 2, or 3), so that every player had equal opportunities. To keep things as simple as possible the games played would end after a certain number of rounds, in order to remove probabilistic aspects of the simulation. A game between two player strategies is completely deterministic and hence only needed to be simulated once.

After simulating all of the games, payoffs were recorded of every possible game between the different playing strategies. Using this information of how games would end, each player strategy was treated as a population. These various playing strategy populations were treated as competing species in an evolutionary environment. For the first round, all populations of playing strategies were equally distributed. A player would increase or decrease in relative population based on the amount of payoff that they earned in a given round. Running these population models made it possible to observe which the "strongest" players were and how the strong players did against each other once the "weak" players died off.

Here are a couple of random examples for seeing this process

A,B,C…X player strategies
Round Robin AAA, AAB, AAC, …AAX… XAA, … XXX

Matrix of Payoffs

AAA = A>0.5, A>0.3, A>0.2
AAB = A>0.6, A>0.3, B>0.1
AAC= A>0.4, A>0.4, C>0.2
…
AAX = A>0.3, A>0.4, X>0.3
…
XAA = X>0.4, A>0.4, A>0.2
…
XXX = X>0.4, X>0.3, X>0.3

In order to maintain the analogy of the competing species, a player's total payoff was a weighted summation based on the probability of a particular set of players being in a given game. For instance, if there were only two players, A and B and 90% of the population was A players, then the chance that the game AAA would take place is 72.9% but only 0.1% for BBB. Thus the payoffs from each game permutation would be multiplied by the probability that that particular game would take place. This total weighted payoff summation was then multiplied by the current population. This could be thought of as analogous to food being divided among different species. After all the populations have been adjusted, the total population of all players is calculated and then the populations for the following round are calculated by taking the percentage each population makes of the total population of all players. The formula for these calculations is as follows:

Pop[i] = percentage of population of player i strategies
Pop[j] = percentage of population of player j strategies
pA(xyz) = payoff A receives in the game where x, then y, and finally z takes their turn

$$\text{Payoff} = \sum_i \sum_j \text{pop}[i] * \text{pop}[j] * (\text{pA}(ijA) + \text{pA}(iAj) + \text{pA}(Aij)) / 3$$

$$\text{pop}[A] = \text{Payoff}*\text{pop}[A]$$
$$\text{totalPop} = \text{Pop}[A]+\text{pop}[B]..\ \text{pop}[X]$$

$$\text{nextRoundPop}[a] = \text{pop}[A]/\text{totalPop}$$

In this model, if a player never receives any payoff, then they will go to population zero (they are dead). As the rounds progress, some player population may drop towards zero (weak players) and other may eventually dominate or reach a population equilibrium with another player.

3. Three-Player Game Results

For the very simplest first experiments, player strategies were designed that essentially only would match a proposal if it satisfied their minimum acceptable payoff and otherwise would propose their minimum acceptable payoff for themselves and split the difference for the other two players. For players of this type, there is essentially a single parameter that determines their behavior- their minimum acceptance level. For the initial testing coalition game, the following was used:

$$w(ABC) = 1$$
$$w(AB) = 0$$
$$w(AC) = 0$$
$$w(BC) = 0$$

This forced the three players to all agree to a proposal or to get a payoff of zero. There is an interesting dilemma among players of this type; on one hand the players do not want to reach a deadlock by having their minimum acceptance level too high, but they do not want to make their acceptance level’s too low, because then they cannot exploit players who would accept outrageous offers. To put this in concrete terms, consider three such players with three different acceptance levels: 0.333, 0.3, 0.4.

When these players are put into competition what happens?

For sake of conceptualization call the 0.3 player generous, 0.4 greedy, and 0.333 egalitarian. When three greedy players play against one another, they cannot reach an agreement, because they each demand 0.4. 3 players*0.4 per player= 1.2 total payoff which is greater than the payoff for the three player coalition. Thus, none of the greedy players would receive a payoff. However, when two generous players and a greedy player play, the greedy player will reject offers made by the generous players and propose 0.4 for himself and propose 0.3 for the two generous players, which they will accept. In this game, the greedy player does better than the generous players. When three generous players play each other, however, the first will propose 0.3 for him and 0.35 for the other two players, resulting in an agreement. Hence, greedy players can exploit generous players while simultaneously hurting themselves when playing against themselves. The egalitarian, however, can exploit the generous players a little, not be exploited by the greedy players, and reach agreements when playing against themselves. For players of this type in our evolutionary environment, the best strategy is to have a minimum acceptance level of exactly one third. Even though a slightly greedy player might do better than the egalitarian in certain populations, greedy players cannot survive alone, because if they were to totally dominate and kill off all other populations, they would then die themselves.

Unfortunately, when trying to introduce more complex strategies, it was very difficult to interpret the results. There seemed to be different effective strategies and results varied greatly from one simulation to another. There could be a number of reasons why this occurred. Perhaps it was that floating point variables allowed for arbitrary precision, perhaps it was that the different variables that were used arrived at different points all of the time because the ratio between variables was the only important factor, not the exact values of individual variables. In hopes of getting more results, the simulation was redesigned so that there was a population of players in the first, second, and third position of every game. That is, there were different player strategies which always took the first turn which competed against each other, as well as similar player strategies for those who went second and third. This was done in hopes of seeing different strategies develop for different positions in the game. This, however, also returned ambiguous results and so an effort was made to simplify the simulations further.

4. Two-Player Game Approach

The next step was to drop to two player games. These games were considerably simpler. Some of the same issues persisted while using floating point variables, so the simulation was modified yet again to be running games which were based on discrete units. For instance, a two player game might be to divide 10 one dollar bills. Six for one, four for the other, or five for each are possibilities, but 5.5 and 4.5 are not. The proposals payoffs must all be integers. Unfortunately, this development came towards the end of the project, so there was not a great deal of time spent investigating these games.

The player strategies in these games were simplified tremendously by making proposals in any round based on the current state of the negotiation (and in some cases previous states). Having strategies this simplistically defined makes it easy to define and track the negotiations that take place. For the simplest game, where two players must decide which will receive a single unit payoff, with no division possible, the results were quite clear. Players that only would propose the one unit for themselves would do well earlier, because no other player could gain more payoff than them, however an entire population of these "greedy" players cannot survive as no payoff is ever received since all the players are "too stubborn." Player doing the best were those that would propose 1 for themselves initially, but would eventually give in and allow the opposing player to have the payoff of one.

5. Conclusion

From the work that was done this summer, a very important lesson to be learned is that it is likely best to start as simple as possible and then start simpler, so that complexity could have been increased over the summer instead of decreased. However, much ground work has been laid for further pursuit into the more complex cases. It is hoped that future work could be done to explore more discrete two player games and hopefully characterize ideal playing strategies for those games. Perhaps with sufficient work the three player game may be implemented as a discrete game as well and hopefully through the evolution of playing strategies it might be possible to characterize more playing strategies and how they might develop.

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7. References

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